Exam 1 Review

Exam 1 details:

- 5-6 pages of questions
- ONLY the Ti-30x IIS Calculator model is allowed (and required)
- Allowed one **hand-written** 8.5 by 11 in. page (double-sided)
- You must show your work on all problems.
- Covers 10.1, 2.1-2.3, 2.5-2.8 3.1-3.3. You should know all the facts and concepts covered in the homework for those sections.
- You have 80 minutes to complete the exam.

Studying Advice:

- Spend 15-30 minutes reviewing all homework.
- Spend 15-30 minutes flipping through 8+ old exams.
- Spend several hours working through several old exams in detail.
- Practice managing your time, never spend more than 10 minutes on a page!

Exam 1 Basic Facts

1. Precalculus: Know all the standard functions, know trig and trig values, know your algebra skills, know circular motion and basic parametric facts.
2. Limit Foundations: Know limit notation, be able to get limits from the picture of a graph, know definitions.
3. Finite Limits: Know techniques ?/0, 0/0, factoring, expanding, conjugate, fractions.
4. Limits at Infinity: Know techniques for ∞/∞, identify known limits, divide top/bottom
5. Continuity and Differentiability: Know what these terms mean!
6. Derivative Foundations: What is $\frac{f(x+h)-f(x)}{h}$, be able to compute this for a given function.
7. The derivative graph: Know the connections between the graphs of $f(x)$ and $f'(x)$.
8. Derivative Rules: Know the sum and coefficient rules, the power rule, the exponential rule, all trig, the product rule and the quotient rule.
9. Applications: Know units of derivative, understand what it represents as a rate, be able to do tangent line questions.

Basic Precalculus:

1. Power/exponent rules.
2. Solving equations (using inverses and the quadratic formula)
3. Understanding what $x = \cos(t)$ and $y = \sin(t)$ gives.
4. Knowing standard trig rules and values (unit circle)
5. Factoring/Expanding/Conjugate
6. Basics of plotting parametric points
7. Functional Notation!

Limits:
First, see my previous postings on the course website reviewing limits.

1. 'Plug in'
2. If it approaches ?/0 and the numerator is not zero, then examine the sign of the expression from both sides ($-\infty$, $+\infty$, or DNE).
3. If it appoaches 0/0, then use factoring, expanding, conjugates, and simplifying fractions to eliminate the zero in the denominator if possible.
4. If it approaches ∞/∞, then divide top and bottom by $1/x^a$ or $1/e^{rx}$ and try to rewrite the expression in terms of known functions.
5. Know the standard precalculus functions, it helps!
6. Know that $\lim_{x\to 0} \dfrac{\sin(x)}{x} = 1$

Derivative Foundations and Basic Facts:

1. What is $\frac{f(x+h)-f(x)}{h}$? What does it represent graphically? What does it represent in an applied problem with units?
2. What is $\lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$? Same questions as above.
3. What are the connections between the graphs of $f(x)$ and $f'(x)$?

Continuity/Differentiability:

1. Know what it means to be continuous at a point. How do we check it?
2. Know what it means to be differentiable at a point. How do we check it?

Derivative Rules

1. What are the sum and coefficient rules?
2. What are the product and quotient rules?
3. What is the derivative of x^n?
4. What is the derivative of a^x?
5. What is the derivative of the 6 trig functions?

Applied Questions:

1. Find the equation for a tangent line given a point on the curve.
2. Find the equation for a tangent line given a point on the line, but off the curve.
3. Set up a problem involving trig or a circle or something from precalculus and do a limit or a derivative.

Math 124 - Winter 2017
Exam 1
January 31, 2017

Name: _____

Section: _____

Student ID Number: _____

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PAGE 3	10	
PAGE 4	10	
PAGE 5	11	
PAGE 6	12	
Total	60	

- There are 6 pages of questions. Make sure your exam contains all these questions.
- You are allowed to use a Ti-30x IIS Calculator model ONLY (**no other calculators allowed**). And you are allowed one **hand-written** 8.5 by 11 inch page of notes (front and back).
- Leave your answer in exact form. Simplify standard trig, inverse trig, natural logarithm, and root values. Here are several examples: you should write $\sqrt{4} = 2$ and $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ and $\frac{7}{2} - \frac{3}{5} = \frac{29}{10}$ and $\ln(1) = 0$ and $\tan^{-1}(1) = \frac{\pi}{4}$.
- Show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.**
- If you need more room, use backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- There may be multiple versions of the exam so if you copy off a neighbor and put down the answers from another version we will know you cheated. Any student found engaging in academic misconduct will receive a score of 0 on this exam. All suspicious behavior will be reported to the student misconduct board.
  DO NOT CHEAT OR DO ANYTHING THAT LOOKS SUSPICIOUS!
  WE WILL REPORT YOU AND YOU MAY BE EXPELLED!
  Keep your eyes down and on your paper. If your TA sees your eyes wandering they will warn you only once before taking your exam from you.
- You have 80 minutes to complete the exam. Budget your time wisely.
  SPEND NO MORE THAN 10 MINUTES PER PAGE!

1. (8 pts) Determine the values of the following limits or state that the limit does not exist. If it is correct to say that the limit equals ∞ or $-\infty$, then you should do so. **In all cases, show your work/reasoning. You must use algebraic methods where available. And explain in words your reasoning if an algebraic method is not available.**

   (a) $\lim_{x\to 3^-} \dfrac{x^2-4}{x-3}$

   (b) $\lim_{t\to 0} \left(\dfrac{2}{t(1+3t)^2} - \dfrac{2}{t} \right)$

   (c) $\lim_{x\to\infty} \dfrac{4x^2 - \sqrt{x^4+2x-1}}{e^{-x}+3x-\sqrt{5x^3+16x^4}}$

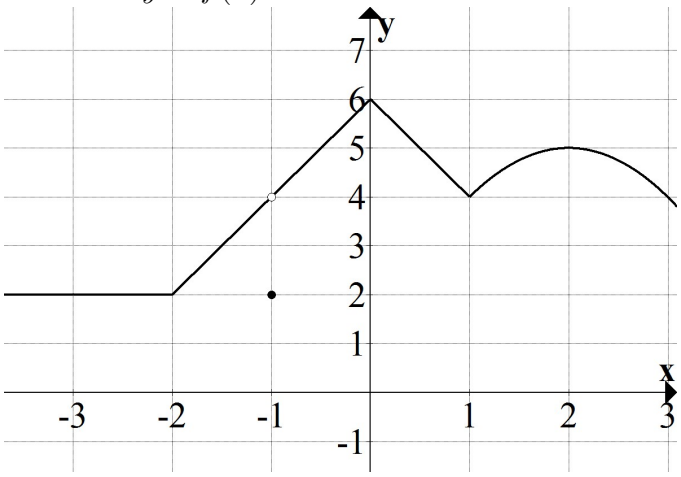
2. (9 pts)

(a) Compute $\lim_{t\to\pi/2} \dfrac{\sin(t) - \sqrt{\sin^2(t) + 4\cos^2(t)}}{3\cos^2(t)}$

(b) Let $y = \dfrac{5}{2x} + \dfrac{4x}{5\sqrt[4]{x}} - \dfrac{4\tan(x)}{x^5}$. Find $\dfrac{dy}{dx}$. (You don't have to simplify)

(c) Let $f(t) = 5te^t\cos(t)$. Find the slope of the tangent line to $f(t)$ at $t = \pi$.

3. (10 pts) Consider the function $y = f(x)$ shown:



Use the graph to estimate/compute the answer to the following:

(a) Find the solution(s) to $f'(x) = 0$.

(b) Name the value(s) of x at which $y = f(x)$ is not differentiable.

(c) Compute $\lim_{x \to -1} \left(f(x) + \dfrac{|x-5|}{\sec(\pi x/6)} + \dfrac{\sin(x+1)}{x+1} \right)$.

(d) If $g(x) = \dfrac{f(x)}{x^2}$, then find value of $y = g'(x)$ at $x = \dfrac{1}{2}$.

4. (10 pts) Consider $f(x) = \begin{cases} bx^2 + 3ax - 10 & , \text{ if } x < 1; \\ ax - b - 2 & , \text{ if } 1 \leq x \leq 3; \\ \dfrac{x^2 - 9}{x - 3} & , \text{ if } x > 3, \end{cases}$ where a and b are constants.

(a) Find the values of a and b that make f continuous everywhere.

(b) Using the values for a and b from part (a), is the function $f(x)$ differentiable at $x = 1$? Clearly say NO or YES, and explain your answer in words. (Hint: Use all our derivative rules to analyze $f'(x)$ near $x = 1$.)

5. (11 pts) A water balloon is thrown upward from a dorm window, it goes up for a bit then ultimately falls down to the ground and *coincidentally* lands near your math instructor. The height of the balloon is given by $s(t) = 80 - 16t^2 + 8t$ where t is in seconds since it was thrown and $s(t)$ is in feet.

   (a) What is the average velocity of the balloon from $t = 0$ to $t = 2$ seconds? (include units)

   (b) Find and *completely simplify* the formula for the average velocity of the balloon from t to $t + h$. That is, find and completely simplify $\dfrac{s(t+h) - s(t)}{h}$.

   (c) Find the instantaneous velocity at the time the balloon hits the ground? (include units)

6. (12 pts) **NOTE: The two questions below are unrelated**.

(a) Find all points (a, b) at which the function $y = \dfrac{x^2}{3x - 6}$ has a horizontal tangent line.

(b) Find all points (a, b) on the curve $y = \dfrac{5x}{3} + 4x^3 - 17$ where the tangent line at (a, b) also passes through the point $(0, 10)$.